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Spin Viscosity Effects in Planar Ferrofluid Couette Flow of Ferrofluids

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Abstract

Previous studies of planar Couette flow of ferro-fluids in a transverse time-independent external magnetic field are extended to include the effects by spin diffusion. We numerically study the modification in the internal rotation of particles in a colloidal ferro-fluid. In particular, we consider a ferro-fluid between two concentric cylinders, and apply a radial magnetic field inversely proportional to the radial variable. We consider the influence of shear effects by assuming a constant magnetic field while varying the rotation rate of the inner cylinder, and the effects of the balance between shear and magnetic stresses by varying the values of the magnetic field while maintaining a fixed rate of rotation for the inner cylinder. We use a continuum model where the Navier-Stokes like conservation of linear momentum equation is coupled to an equation governing the angular momentum of isotropic particles and a Magnetization equation that describes the behavior of magnetic particles. We demonstrate that the internal rotation undergoes a directional reversal in the channel and study the relationship between the magnitude of the magnetic field and the location of the sign-change. Moreover, we show that there is an inverse relationship between fluid spin and where the change in direction of rotation takes place. Previous research has demonstrated that the direction of internal rotation of particles of ferrofluids flowing in a channel with parallel plates changes at some point of the channel. Our study aims to establish how this point in the channel is related to the applied magnetic field and the spin viscosity of the the ferrofluid.

Spin Viscosity Effects in Planar Ferrofluid Couette Flow of Ferrofluids

A THESIS

Submitted in partial fulfillment of the requirements
for the degree of Masters in Pure and Applied Mathematics

by

Lucía Cataldo Ottieri
Montclair State University
Montclair, NJ

2017

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MONTCLAIR STATE UNIVERSITY

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Lucía Cataldo Ottieri

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College of Science and Mathematics

Department of Mathematics

Thesis Committee:

[redacted]

Dr. Arup Mukherjee
Thesis Sponsor

[redacted]

Dr. Bogdan Nita
Committee Member

[redacted]

Dr. Ashuwin Vaidya
Committee Member

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Chapter 1

Introduction and Outline

Ferrofluids are *magnetic liquids* where the magnetic particles in the carrier fluid may not be aligned with the direction of the local magnetic field. This response behavior of ferrofluids play a significant role in their applications. Section 2.1 provides a brief introduction to magnetism and ferrofluids, while we present both current and future applications of these magnetic liquids in Section 2.2. Since ferrofluids are not found in the nature, we include a brief explanation about their synthesization and the behavior of magnetizable particles in response to applied magnetic changes in Section 2.3.

The local magnetization of a ferrofluid under the influence of an applied field plays a key role in determining the nature and flow properties of the fluid. Some ferrofluid magnetization effects relevant to the present study are presented in Sections 2.4 and 2.5.

Ferrofluid flows can be mathematically modelled using the continuum theory for fluids with micro-structure. The basic equations governing ferrofluid behavior and flow are the balance of linear and angular momenta coupled with a magnetization equation. Shliomis [16] formulated the magnetization equation for ferrofluids. A detailed overview of ferrofluid behavior can be found in [12] or in the survey article [10] written by Rinaldi and co-authors. For this study, we consider the flow of a ferrofluid in the annular gap between two concentric cylinders of infinite length subject to a radial time-independent magnetic field. We provide a specific description of the geometry in Section **??** and present the governing equations in Section 3.1. The coupled system of equations for ferrofluid flow reduce to a boundary value problem for a system of ordinary differential equations. We use standard routines in MATLAB® to numerically study the effects of the shearing rate applied via an internal rotation of one cylinder while the other is stationary, as well as the effect of changing the strength of the applied magnetic field. A brief outline of our numerical procedure is given .

For fluids without microstructure, the velocity field is sufficient to determine the kinematics. In particular, the symmetric part of the velocity gradient contains information required to determine the normal and shear strains, while the vorticity (or the related spin tensor which is the skew-symmetric part of the velocity gradient) measures the rates of rotation of line elements in an average sense. However, for ferrofluid flow, couple stresses can play an important role in the bulk kinematics. An example of the importance of couple stresses in ferrofluids is the discrepancy between experimental observations and numerical computations for spin-up flow in an annular gap in a rotating magnetic field. Typically, the mathematical models used in the analysis assume that the spin viscosity is zero. Assuming a non-zero value of the spin viscosity but a uniform magnetic field in the annular gap, Rosensweig and co-authors [13] obtained a flow direction for spin-up flow opposite to experimental observations. Chaves et. al. [3] resolved this discrepancy by determining an asymptotic solution to a model which retains a small non-zero value of the spin viscosity. The non-zero spin assumption leads to a non-uniform magnetic field in the gap. For non-zero spin viscosity, Chaves et. al. obtain co-rotation of the fluid near the outer cylinder and counter-rotation close to the inner cylinder wall. The transition occurs at an intermediate position in the annular gap, and the numerical values qualitatively match experimental observations. Additionally, the work of Rosensweig [14] and Jothimani and Anjali Devi [5] highlights the importance of internal rotations on the performance of magnetic fluid rotary seals and bearings. Both assume uniform applied magnetic fields in the ferrofluid and ignore the effects of transport of angular momentum along a gradient of spin (equivalent to zero spin viscosity assumption). A brief overview of these results is given in Chapter 4.

In this thesis, by considering ferrofluid in an annular gap under the effect of an time-independent external field and retaining a small non-zero value of the spin viscosity, we show that the internal rotation of the ferrofluid changes sign at an intermediate position in the gap. Although our study does not show a change in direction of the bulk flow as obtained for spin-up, the directional transition of the internal rotation may have a significant effect on some applications. We present our results and discussions in Chapter 5.

Chapter 2

Fundamental properties, applications, and Synthesis of Ferrofluids

2.1 Magnetism and Ferrofluids

Magnetism, by definition, is a property observed in magnets and electric currents of being able to exert actions such as attractions or repulsions to other objects. These actions often interact on certain metallic elements like iron, nickel, cobalt, and their alloys. Magnetic interactions can also be observed in rare earth elements such as gadolinium. By the 1900s, scientists such as Wilson thought about applying magnetism over magnetic liquids, as mercury, to control their flow, see [17] for further discussions. It was during the 20th Century that this interest increased and multiple studies followed. NASAs focus shift to the study of ferrofluids began due the advantage of controlling the flow of fluids in a zerogravity environment. Steven Papell pioneered studies and ultimately developed non-metallic fluids with magnetic characteristics during the 1960s. Ferrofluid research includes the study of the fundamental properties, synthesis, and applications. Using old techniques, these magnetic fluids lost the capacity of flow in the presence of stronger magnetic fields. The synthesis of ferrofluids has improved, and today, ferrofluids retain their fluid behavior even under the effect of strong magnetic fields [15].

The change in physical properties observed in ferrofluids are not only derived from varying the strength of the applied magnetic field, but also from the possibility of tailoring the ferrofluid depending what would be their use.

In the quest of developing more understanding and uses for ferrofluids, different disciplines are involved. The mathematical study of the behavior of these fluids is based on a continuum model for fluids with micro-structure. In particular, the generalization of the Navier Stokes like equation, for the balance of linear momentum is coupled to a balance another for the time development of Magnetization. Liu et al. [6] demonstrated the efficiency and advantage of using ferrofluids in the delivery of drugs in cancer treatments. They proposed a mathematical model with the following considerations: the better ferrofluid in this situation would be made of water in order to mix well with the blood, the channel through which the ferrofluid moves is rigid, and the ferrofluids velocity profile should be parabolic. This study provides an example of the application of ferrofluids and its mathematical models in the field of biology and medicine. The work of many disciplines, jointly, has made the interest on these fluids more broad. Although the comprehensive understanding of these liquids is still under way, there are applications already in use and show promise for future application [12]. Some of the current applications are presented in the next section.

2.2 Applications

Imposing magnetic fields on ferrofluids allows for the directional control of their flow, placement of the ferrofluid, and fluctuation of their temperature and viscosity. These characteristics have enabled the use of ferrofluids in: acoustics, sealing, pressure control, medicine, clutches, brakes and dampers.

Acoustics: The temperature of ferrofluids is capable of changing when a magnetic field is applied to them, and due to this characteristic, they are used in acoustics. The voice coil becomes hot and the heat is transferred to the loudspeakers. Ferrofluids obey the Curie Law, which states that the hotter a magnetic material is, the less magnetic it becomes. When a magnet is placed in a coil, it will attract the hottest surrounding ferrofluid. Consequently the circular flow is produced will have a cooling effect, which improves the performance of the loudspeaker [2, 12].

Sealing: In rotating shafts, ferrofluids are placed in the cavity where an axel rotates. The walls of the cavity act as the sides of a magnet which generates a field making the ferrofluids stay in place. Since in these cases the liquid used is an oil-based ferrofluid, the fluid also helps with the friction in between metal parts, while acting as a sealer in between the sides of the shaft. The sealing works from vacuum to overpressure settings. The gradient of pressure is carried on for several stages of ferrofluids seals, each of which hold a fraction of the total pressure desired. Thus, ferrofluids find use in electronic, sealing-particularly, when is necessary to bring rotational power in a vacuum and clean environment. They also are used for sealing hard disc drives, semiconductors, rotating X-ray tubes, and certain engines [12].

Control of pressures: Since ferrofluids are incompressible, when used in between two gases or two environments of different pressures, ferrofluids will allow these gases and/or environments to keep their initial pressure values as reported in [1].

Medicine: The applications in the medical field are in the experimental stage. Since ferrofluids can be easily located in a place using magnetics fields, they have used in experimental retinal surgeries [15]. In the fight against cancer, ferrofluids have been used due to their capability to be easily positioned. By increasing the temperature of tissue near target areas the transportation of medicines to specific locations (difficult to access with current medicines) within the body is made easier by ferrofluids [15].

Clutches, brakes, dampers: Ferrofluids have been tested in clutches, brakes and dampers due to their capability of changing their viscosity when a magnetic field is applied. The devices introduced above are simple oscillation systems, in which ferrofluids act as a magnetic fluid damper, making the resonance of the movement damped out.

2.3 Synthesis of Ferrofluids

Ferrofluids are colloidal suspensions of finely divided magnetizable particles in a continuous medium. A valid medium can be water, different kind of oils, and kerosene. The fluid medium could also be a metallic liquid such as tin, gallium and mercury whose electrical and thermal conductive properties can be taken as advantage [9]. One problem of having particles in suspension is to maintain them in a stable dispersion. This concern has been solved for different mediums, but not for metallic base ferrofluids [9]. Since reducing friction between metal parts is an important role for ferrofluids in industrial applications, many ferrofluids are oil based. In all the cases, the particles in the suspension are finely and evenly divided, giving ferrofluids an opaque look.

The magnetizable particles in suspension can be prepared mechanically or chemically to arrive to sizes to the order of 100Å. When using mechanical methods, bigger particles are reduced in size in a process that takes several hours. The chemical process consists of making a solution to precipitate nano-size particles. Whichever the process used for creating nano-size particles, scientists look for the resultant ferrofluids to be stable. Therefore, it is important for the particles not sediment due to gravitational or magnetic forces, or to agglomerate or coagulate.

In order for the particles to not sediment, their thermal energy should be higher than the energy of the particles in the gravitational field and/or magnetic field. For typical values of practical magnetic fields, the aprticle diameter should be between 5 and 10 nm.

Due to Van der Walls forces, particles in the ferrofluid tend to agglomerate, and the fluid looses its key characteristic of liquid flowability [9]. A surface-active agent surfactant [7]is added to enable the ferrofluid to retain this fundamental property. This surfactant coating provides a repulsive energy between particles an a synthesized ferrofluid, and prevents agglomeration. It is important that this coat and the carrier fluid are not similar, because in such a case both will mix and the effect of the surfactant will be inhibited.

In these fluids, we have particles in suspension of different size, and for this reason they act differently when a magnetic field is applied to them. We will describe the different processes of magnetization that particles have, due to their different sizes.

2.4 Relaxation of Magnetization

An important characteristic of ferrofluids is their capability of being magnetized due to the application of a magnetic field. The process through which a variable changes over time is called the relaxation of the variable. Since certain characteristics of these fluids such as temperature and viscosity change when the applied magnetic field is altered, it is important to understand how the magnetizable particles behave due to the relaxation of magnetization. The changing of the imposed magnetic field in some cases causes the particles to rotate, and in other cases the magnetic vector inside the particle rotates [12]. The process through which the entire particle rotates is called Brownian Relaxation. On the other hand, the time for the magnetic field inside the particle to change is called the Néel Relaxation.

The duration of these mechanisms differs depending, primarily, on the volume of a given particle. The mechanism preferred by the particle is that of shortest time it takes to complete. If the time required for a Brownian Relaxation is less than that of a Néel Relaxation, then a Brownian Relaxation is favored. Particles with diameter less than 15 nm will favor the Néel mechanism, while larger particles prefer the Brownian relaxation mechanism. When the ferrofluid is synthesized, the magnetizable particles end up having different sizes. This implies that, in the same ferrofluid subject to an external magnetic field, both relaxation times described above occur simultaneously.

2.5 Equilibrium of Magnetization

When a ferrofluid is not under the influence of a magnetic field, the particles do not have a particular orientation. When a magnetic field is applied over the fluid, the particles in suspension tend to align in the direction of the magnetic field. For the weak magnetic fields, the thermo energy of the fluid is strong enough to overcome the inclination of the particles to orient along the field's direction. However, with increasing applied field strength, the orientation is preferred. The size of the particles in suspension will decide how this alignment will be, and the equilibrium is reached following the Brownian and Néel mechanisms. When all the particles are aligned with the external magnetic field applied, we will say the saturation value of the field has been reached and the system is in equilibrium.

In the case we are studying, though, the vector magnetic field applied and the magnetic vector of the particles are not aligned. The fluid is in motion and due to the effect of the fluid's shear there is a transport of angular momentum by the diffusion of spinning magnetic particles down a gradient of spin [14].

Chapter 3

Governing Equations and Geometry

Ferrofluids are synthesized colloidal mixtures of magnetic nano-particles in a non-magnetic carrier fluid. The nano-particles are coated with a surfactant which prevents particle agglomeration while Brownian motion prevents them from settling under gravitational forces. Contrary to magneto-rheological fluids, the surfactant coating in a synthesized ferrofluid allows it to retain liquid-like properties under high-gradient external magnetic fields. In addition to the translational velocity and internal rotation needed to describe the kinematics of fluids with micro-structure, the magnetization in a ferrofluid is governed by the relaxation equation derived by Shliomis [16].

3.1 Governing equation for Ferrofluids

The Navier-Stokes equations governing the flow of Newtonian fluids constitute a mathematical model made up of a set of non-linear differential equations that describe the movement of viscous fluids. The model can be viewed as a continuum theory generalization of Newton's second Law $F = m \cdot a$ by considering a differential volume over which a normal and a shear stress are applied. The fluid is assumed to satisfy the continuum hypothesis (viscosity, density, pressure, temperature are well defined in a differential volume and do not change with the rate of flow). Instead of the traditional description of mass, the density of the object is introduced as the mass per unit volume. The shear stress is defined as the component of stress co-planar with the material cross section is used as a external force [4].

For ferrofluids, we consider a point in the fluid and a differential volume around it and define $\mathbf{u} = \mathbf{u}(\mathbf{x}, t)$ as the translational velocity, and $\boldsymbol{\omega} = \boldsymbol{\omega}(\mathbf{x}, t)$ as the internal rotation of the point $\mathbf{x}$ at any time t. The viscosity of a ferro-fluid causes a lag in the magnetization $\mathbf{M}$ in relation to the applied magnetic field $\mathbf{H}$. When these fields are not collinear, a torque $\mathbf{I} = \mu_0 \mathbf{M} \times \mathbf{H}$ develops, and causes the viscous stress tensor to have an anti-symmetric part. In this situation, a balance of angular momentum equation is needed to capture novel flow phenomena that may arise. The *linear momentum balance* that governs the flow of a ferrofluid is

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{u}\right)=\nabla\cdot\mathbb{T}+\mathbf{f}, \tag{3.1}$$

where ρ is the fluid density, $\mathbf{f}$ represents all external forces applied to the fluid and equals $\mu_0\mathbf{M}\cdot\nabla\mathbf{H}$, and $\mathbb{T}$ is the Cauchy stress tensor defined as

$$\mathbb{T}=-p\mathbb{I}+\eta(\nabla\mathbf{u}+\nabla\mathbf{u}^T)+\zeta\mathbb{E}\cdot(\nabla\times\mathbf{u}-2\boldsymbol{\omega}) \tag{3.2}$$

The parameter η is the coefficient of shear, ζ represents the vortex viscosity, $\mathbb{I}$ is the identity matrix, and $\boldsymbol{\omega}$ the velocity of spin introduced above. Equation(3.1) is complemented with the *Angular Momentum* balance equation

$$\rho I\left(\frac{\partial \boldsymbol{\omega}}{\partial t}+\mathbf{u}\cdot\nabla\boldsymbol{\omega}\right)=\nabla\cdot\mathbb{C}+\mathbf{T}+\mathbf{I} \tag{3.3}$$

where I is the moment of inertia density, $\mathbf{T}=-\mathbb{E}:\mathbb{T}$ the antisymmetric vector of the Cauchy stress, and the body couple density $\mathbf{I}=\mu_0\mathbf{M}\times\mathbf{H}$. Also, the couple stress tensor $\mathbb{C}$ defined as:

$$\mathbb{C}=\eta'(\nabla\boldsymbol{\omega}+\nabla\boldsymbol{\omega}^T) \tag{3.4}$$

where η' represents the shear coefficient of spin-viscosity. For a complete description of the kinematics of a ferrofluid, we need to add the conventional *Magnetization equation* for an incompressible fluid derived by Shliomis [10]

$$\frac{\partial \mathbf{M}}{\partial t}+\mathbf{u}.\nabla\mathbf{M}=\boldsymbol{\omega}\times\mathbf{M}-\frac{1}{\tau}(\mathbf{M}-\mathbf{M}_0). \tag{3.5}$$

In Equation(3.5), $\mathbf{M}_0$ is the equilibrium magnetization, and τ is the effective magnetization relaxation time. The magnetic moment in each of the nano particles in suspension will align with the magnetic field $\mathbf{H}$ externally applied during the relaxation of magnetization. In case the magnetic momentum in the particle rotates, the time for the equilibrium is called the Néel relaxation time and it is represented by τ_N. On the other hand, when the magnetic moment inside the particle rotates, the time for the equilibrium is called the Brownian relaxation magnetization and it is represented by τ_B. Since we are considering a continuous fluid, we consider both times and we define the total magnetic time constant [10] as

$$\frac{1}{\tau}=\frac{1}{\tau_B}+\frac{1}{\tau_N}. \tag{3.6}$$

The coupled system of differential Equations (3.1),(3.3), and(3.5) together with equations of continuity models the general flow of ferrofluids under the influence of an external field $\mathbf{H}$. It is typical to assume incompressibility which leads to $\nabla\cdot\mathbf{u}=0$ and $\nabla\cdot\boldsymbol{\omega}=0$. These equations also have to be supplemented by boundary and initial values for the translational velocity $\mathbf{u}$, spin velocity $\boldsymbol{\omega}$, and magnetization $\mathbf{M}$.

For time-independent flows, the right hand side of Equation(3.3) coupled with Equation(3.2) reduces to a balance between *spin excess* $\nabla\times\mathbf{u}-2\boldsymbol{\omega}$, the spin-diffusion $\eta'\Delta\boldsymbol{\omega}$, and the magnetic torque $\mu_0\mathbf{M}\times\mathbf{H}$

$$\overbrace{\nabla\times\mathbf{u}-2\boldsymbol{\omega}}^{spin\ execess} = \overbrace{\eta'\Delta\boldsymbol{\omega}}^{spin\ diffusion} + \overbrace{\mu_0\mathbf{M}\times\mathbf{H}}^{torque} \tag{3.7}$$

When the diffusion of spin is ignored ($\eta' = 0$), Equation(3.7) reduces to an algebraically equation and can be solved to find an approximate value of $\boldsymbol{\omega}$ provided the magnetization $\mathbf{M}$ is known. Further, for $\eta' = 0$, boundary conditions are not needed for the spin velocity $\boldsymbol{\omega}$, while for $\eta' \neq 0$, boundary conditions on spin have to be specified for solving equation(3.7). Appropriate boundary conditions for spin (or vorticity for regular fluids) are a subject of debate, and various choices are possible, depending on the nature of the interaction of particles and the boundary. Ignoring the diffusion of spin in the governing equations can generate effective solutions of ferrofluid flow for many applications. However, as mentioned in the introduction , retaining a small but non-zero spin viscosity can play a crucial role in other models (for example, spin-up flow).

Summarizing, the equations governing the flow of ferrofluids are

$$\rho\left(\frac{\partial\mathbf{u}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{u}\right)=\nabla\cdot\mathbb{T}+\mathbf{f}, \tag{3.8}$$

$$\rho I\left(\frac{\partial\boldsymbol{\omega}}{\partial t}+\mathbf{u}\cdot\nabla\boldsymbol{\omega}\right)=\nabla\cdot\mathbb{C}+\mathbf{T}+\mathbf{I}, \tag{3.9}$$

and

$$\frac{\partial\mathbf{M}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{M}=\boldsymbol{\omega}\times\mathbf{M}-\frac{1}{\tau}(\mathbf{M}-\mathbf{M}_0) \tag{3.10}$$

where $\mathbf{u},\boldsymbol{\omega}$ and $\mathbf{M}$ are unknowns. The external applied field $\mathbf{H}$ is specified and the equilibrium magnetization $\mathbf{M}_0 = \chi_0\mathbf{H}$ is computable. The parameters τ,η,η',ζ, and *rho* are ferrofluid specific. Thus, under appropriate boundary and initial conditions that depend on the geometry and physics, the coupled system of non-linear differential Equations (3.8), (3.9), (3.10) can be solved to obtain the translational velocity $\mathbf{u}$, the spin viscosity $\boldsymbol{\omega}$, and the magnetization $\mathbf{M}$ for any ferrofluid.

3.2 Plane Couette Cylinder

Ferroluids have been studied and tested in different environments using diverse devices, one of those is the plane Couette Cylinder, Fifure(3.2). This equipment consists of two concentric cylinders rotating with different velocities with the fluid in the annular region between them. Depending what we are experimenting, both cylinders can rotate or one can be fixed while the other rotates. After certain amount of time the flow arrives to a steady state where variables do not change when time changes, and this is the period in where our study was made. This device has been used to

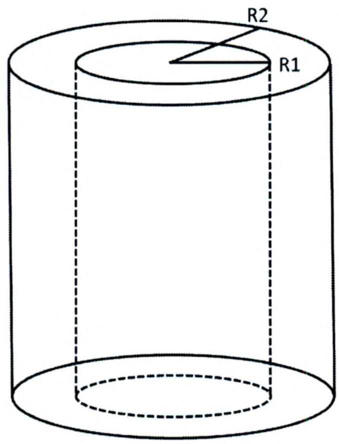

Figure 3.1: *PlaneCouette Cylinder with interior radius R_1, exterior radius R_2, and gap length* $R = R_2 - R_1$

study steady flows and flow bifurcations which has helped in understanding the effect of the spin boundary layer [4].

3.2.1 Viscosity of Ferrofluids

Viscosity is the property of fluids to resist the flow due to the friction at molecular level as well as the friction produced between the different layers in the liquid. As a result of having particles in suspension, ferrofluids as a whole posses more mechanisms for producing viscosity than the liquid carrier by itself. Considering the translational flow, a deformation exists between two consecutive layers of fluid, and the shear viscosity coefficient, η, is the constant of proportionality between the deformation and the stress in between the layers. Similar deformations exist if we consider the spin and the vortex of the fluid. Consequently we define η' the coefficient of spin viscosity as the constant of proportionality between the spinning layers of the fluid. Also, ζ is the coefficient of vortex viscosity and is the constant of proportionality between the deformation between layers of the fluid and the stress due to the vorticity. The ratio between viscosity (η) and vortex viscosity (ζ) in ferrofluid is proportional to the volume fraction of particles in the suspension (Φ) via

$$\frac{\zeta}{\eta} = \frac{3\Phi}{2}. \tag{3.11}$$

3.2.2 Vorticity

The vorticity is a measure of the local rotation of a fluid. Applying the "right hand rule", the direction of the thumb would represent direction and magnitude of the vector vorticity. The other four fingers would follow the vector velocity of the fluid. The faster the translational velocity is, the stronger the vorticity vector is. Mathematically speaking, the vorticty is the curl of the velocity field ($\vec{\mathbf{u}}$), and is given by $\nabla \times \vec{\mathbf{u}}$.

We studied ferrofluids under the effect of a radial magnetic field applied in a plane Couette cylinder. The numerical model for studying this phenomenon was obtained by applying the Navier-Stokes equations to the fluid. After made certain assumptions we used the *bvp4c* routine of MATLAB® to solve the system of differential equations and generate graphs. In the next chapter we described what a plane Couette cylinder is, we explain briefly the Navier-Stokes equations and present the routine used to produce the graphs.

3.3 MATLAB® code

Once our problem was defined and conditions were set, we wrote a program using the routine bvp4c of MATLAB®, which solves boundary value problems for ordinary differential equations. The routine needs an interval to operate as well as two boundary conditions. The interval we used was the distance in between the plane Couette cylinder, parameterized as [0,1]. We imposed no-slip boundary conditions for teh translational velocity $\mathbf{u}$ at the rotating inner cylinder and the stationary outer one, while we assumed a vanishing anti-symmetric stress condition $2\boldsymbol{\omega} = \nabla \times \mathbf{u}$ for the spin velocity $\boldsymbol{\omega}$ at both boundaries. We solved the system of differential equations obtained, for small but non vanishing values of a parameter ϵ to understand the behavior of the solution when $\epsilon \longrightarrow 0$. This parameter ϵ is linearly proportional to the spin viscosity η'. Our work is based on the flow of ferrofluids confined between two parallel cylinders under the effect of an external radial magnetic field. In particular, we studied the case of a ferrofluid in a plane Couette Cylinder, and we extended the analysis of two similar studies where other researchers studied ferrofluids in similar geometries.In the next chapter we summarize these works [5] and [14] and their conclusions. Although, the findings in [5] and [14] are diffent form ours, the contribution of non zero spin viscosity in ferrofluids flow is a common theme.

Chapter 4

Internal Rotations and their effects on Applications

Our study focuses in how viscosity affects the spin in a planar ferrofluid Couette flow. In this chapter, we will present 2 studies, [5] and [14], that are relevant to ours. Although the setup of the model is different than what we did, in both cases the authors consider the spin viscosity η' to be different than zero, and then studied the angular velocity of the ferroifluid. We also reviewed another case [3] where the consideration of non-zero spin viscosity is relevant. In this study, the authors related the non-zero spin viscosity with the change in direction of the trasnlational velocity of the ferrofluid.

4.1 Ferrofluid in annular gap in a rotating magnetic field

Chaves et al [3] examined the flow of a ferrofluid in a plane Couette cylinder by considering the effects of the spin viscosity. The authors applied a rotating magnetic field on the outsider cylinder of the device. They reported that, the ferrofluid's layers close to the outer cylinder rotated in the same direction of the rotating magnetic field applied on the walls. The fluid close to the inner cylinder rotated in the opposite direction of the magnetic field. The relevance of this case is the fact that, considering a non-zero spin viscosity a change in the direction of the fluid's flow was reported.

4.2 Magnetic Field Effects on a Ferrofluid

S. Jothimani and Anjali Devi [5] considered a ferrofluid flowing between two parallel and rigid plates, the upper plate was moving at a constant rate while the lower plate was stationary. The sketch for the setting of their work is shown in Figure 4.2:

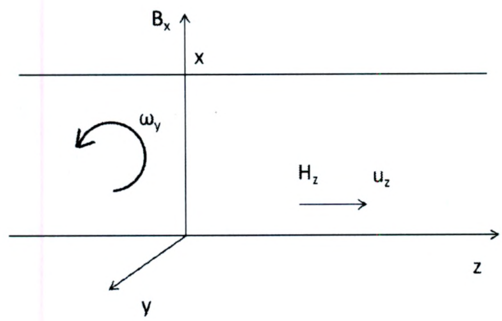

Figure 4.1: *Geometry of the problem presented by Jothimani and Devi, who used a rectangular system of coordinates. A magnetic flux* $\mathbf{B}_0$ *was applied perpendicularly and sinusoidal magnetic field* $\mathbf{H}$ *was imposed on the ferrofluid*

The imposed sinusoidal magnetic field $\mathbf{H}$ and magnetic flux $\mathbf{B}_0$ was applied over the Couette flow were spatially uniform and the authors considered the steady state and only the planar flow of the fluid, as we did. The velocity of the fluid was defined as $\mathbf{u}$, and the spin velocity $\boldsymbol{\omega}$ is considered only perpendicular to the direction of the translational velocity $\mathbf{u}$.

The authors also took into account that the fluid is not compressible which result in the conditions:

$$\nabla \cdot \mathbf{u} = 0 \quad \text{and} \quad \nabla \cdot \boldsymbol{\omega} = 0$$

on the translational and spin velocity.

Under these assumptions, the equations for balance of linear and angular momentum for ferofluids are

$$\rho \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} \right] = -\nabla p + \mathbf{f} + 2\zeta \nabla \times \boldsymbol{\omega} + (\zeta + \eta)\boldsymbol{\omega}^2 \mathbf{u} - \rho g \hat{i}, \tag{4.1}$$

and

$$I \left[\frac{\partial \boldsymbol{\omega}}{\partial t} + (\mathbf{u} \cdot \boldsymbol{\omega})\boldsymbol{\omega} \right] = \mathbf{T} + 2\zeta(\nabla \times \mathbf{u} - 2\boldsymbol{\omega}) + \eta' \Delta^2 \boldsymbol{\omega}, \tag{4.2}$$

where ζ is vortex viscosity, η the dynamic viscosity, η' coefficient of spin-viscosity ρ is the mass density, p pressure, and I is the momentum of inertia density. The magnetic force density was defined as $\mathbf{f} = \mu_0 \mathbf{M} \cdot \nabla \mathbf{H}$.

The imposed magnetic field $\mathbf{H}$ made the fluid to rotate with certain angular velocity. The authors assumed that the velocity of the fluid next to the fixed plate is zero, but other layers of the fluid are in movement. Between the layer assumed fixed and the layer in movement, there exists a deformation proportional to the shear stress, and the viscosity of the fluid is the constant of proportionality.

Similarly, fluid spin results in a spin-friction between consecutive spinning layers of ferrofluids. The constant of proportionality between this deformation and the spin stress is the shear coefficient of spin-viscosity η', a parameter the authors used to classified the possible solutions of the problem.

Using non-dimensional variables, the Equations (4.1) and (4.2) reduce to

$$\frac{1}{2}(\tilde{\zeta}+\tilde{\eta})\frac{d^2\tilde{\mathbf{u}}_z}{d\tilde{x}^2}+\tilde{\zeta}\frac{d\tilde{\boldsymbol{\omega}}_y}{d\tilde{x}}-\frac{\partial\tilde{p}'}{\partial\tilde{z}}=0, \tag{4.3}$$

$$\eta'\frac{d^2\tilde{\boldsymbol{\omega}}_y}{d\tilde{x}^2}-\tilde{\zeta}\left(\frac{d\tilde{\mathbf{u}}_z}{d\tilde{x}}+d\tilde{\boldsymbol{\omega}}\right)+<\tilde{T}_y>=0. \tag{4.4}$$

When solving the previous system, they considered the possibility of having zero and non-zero spin-viscosity η', separately. The authors pondered that from a physical point of view, a small but non zer value of the spin viscosity η' is realistic. They solved the coupled system of Equations (4.3) and (4.4) using a zero magnetic torque density $<T_y>=0$, and a zero pressure gradient $\frac{dp'}{dz}=0$. The torque $<T_y>$ is a measure of the force that makes an object rotate. As we can see in the Fig 4.1, the magnetic flux $\mathbf{B}_0$ produces a torque only in the direction of z, the direction of the flow. Considering zero magnetic torque density, the system of the Equations (4.3) and (4.4) reduces to

$$\frac{1}{2}(\tilde{\zeta}+\tilde{\eta})\frac{d^2\tilde{\mathbf{u}}_z}{d\tilde{x}^2}+\tilde{\zeta}\frac{d\tilde{\boldsymbol{\omega}}_y}{d\tilde{x}}-\frac{\partial\tilde{p}'}{\partial\tilde{z}}=0, \tag{4.5}$$

and

$$\eta'\frac{d^2\tilde{\boldsymbol{\omega}}_y}{d\tilde{x}^2}-\tilde{\zeta}\left(\frac{d\tilde{\mathbf{u}}_z}{d\tilde{x}}+d\tilde{\boldsymbol{\omega}}\right)=0. \tag{4.6}$$

Once this system of equations was set, the authors analyzed 3 different cases considering an axial magnetic field, a transverse magnetic field, and a rotational magnetic field. They observed that in all cases, once the system of non-dimensional differential equations expressed above are coupled while the spin viscosity is taken as non-zero, the spin of the fluid changes direction at some point in the channel.

4.3 Partial Rotation in Ferrofluids

In his work [14], Rosensweig considered a Couette flow with an external time-varying uniaxial magnetic field. The author worked with a Cartesian system where the velocity of the fluid was horizontal and the magnetic field was applied perpendicular to the direction of the flow. When a magnetic field is applied, the ferrofluid as a whole will align with the magnetic field. The viscous shear coefficient represents the constant of proportionality between stress and strain. Similarly, in between the spinning layers will exist a deformation and the constant of proportionality between this deformation and the stress of spin is the coefficient of spin viscosity. Since the value of the viscosity increases when a magnetic field is applied, the shear also will increase. Therefore, the vector of the magnetized fluid will be not aligned with the vector magnetic field applied externally. This discrepancy between the alignments in Rosensweig's work was measured by Θ as seen in Figure 4.2.

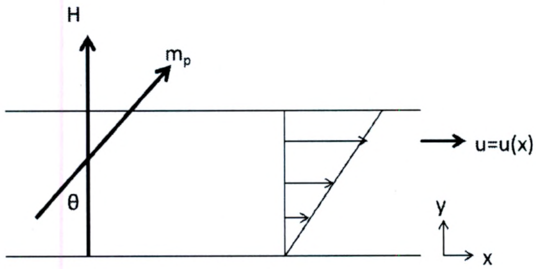

Figure 4.2: *Shear profile of a ferrofluid when a magnetic field* $\mathbf{H}$ *is applied the upper palte moves at a uniform speed while the lower plate is fixed. The variable* θ *measures the discrepancy between the vector magnetic field* $\mathbf{H}$ *and the magnetization* $\mathbf{M} = \mathbf{H} + \mathbf{9m_p}$

In his work, Rosensweig considers the equation of conservation of internal angular momentum for a magnetic fluid:

$$\rho \frac{Ds}{Dt} = \rho \mathbf{G} + \nabla \cdot \mathbb{C} - \mathbf{A}, \tag{4.7}$$

where s is the internal angular momentum per unit mass of magnetic fluid, $\mathbf{G}$ is magnetic body couple density per unit mass, $\mathbb{C}$ is the surface couple stress tensor, and ρ is the density of the fluid. $\mathbf{A}$ is the vector of the antisymmetric part of viscous stress tensor and represents the conversion rate of angular momentum between external and internal forms. Since the work considered the steady case, s does not change with time and therefore $\frac{Ds}{Dt}$ is zero. The term $\nabla \cdot \mathbf{C}$ represents transport of angular momentum by the diffusion of spinning magnetic particles down a gradient of spin, but numerical estimate indicates it is negligible. Rosensweig also defines

$$\rho \mathbf{G} = \mu_0 \mathbf{M} \times \mathbf{H} \tag{4.8}$$

and

$$\mathbf{A} = 4\zeta(\boldsymbol{\omega}_f - \boldsymbol{\omega}_p) \tag{4.9}$$

where μ_0 is permeability of free space, $\mathbf{M} = \eta_0^{-1}\mathbf{B} - \mathbf{H}$ is magnetization of the magnetic fluid, ζ the vortex viscosity, $\boldsymbol{\omega}_f = \boldsymbol{\Omega}$ is the fluid rotational rate, $\boldsymbol{\Omega} = \frac{1}{2}\nabla \times \mathbf{u}$ is the vorticity, $\mathbf{u}$ the vector velocity, and $\boldsymbol{\omega}_p$ the angular spin rate which is proportional to s. Substituting Equations(4.8) and (4.9) in the the equation of conservation of internal angular momentum (4.7) we obtained

$$0 = \mu_0 \mathbf{M} \times \mathbf{H} + 4\zeta(\boldsymbol{\omega}_f - \boldsymbol{\omega}_p). \tag{4.10}$$

This equation is coupled to the relaxation equation

$$\frac{D\mathbf{M}}{Dt} = \boldsymbol{\omega}_p \times \mathbf{M} - \frac{1}{\tau}(\mathbf{M} - \mathbf{M}_0) = 0 \tag{4.11}$$

and the coupled system can be solved when $D\mathbf{M}/Dt = 0$ to obtain an algebraic equation for M, where $bfM = \mathbf{M}_0 + m$. The time of relaxation magnetization depends on the size of the particles. Since in a ferrofluid there are particles of different size, these processes would happen simultaneously. Rosensweig, considered the ferrofluid as a continuous fluid, therefore the time of relaxation magnetization was taken as an average of the two times that take place, and the value was represented by τ.

The vector m_x satisfies the algebraic equation

$$\frac{m_x}{M_0}(\Omega\tau)^2 - \left[2\left(\frac{m_x}{M_0}\right)^2 + 1\right](\Omega\tau) + \left(\frac{m_x}{M_0}\right)^3 + 2\left(\frac{m_x}{M_0}\right) = 0 \tag{4.12}$$

The author considered ferrofluids with: $\eta_0 M_0 = 0.06$ Tesla, $\eta_0 H_0 = 1.5$ Tesla, $\tau = 2 \times 10^{-5}$ seconds, and $\zeta = 0.2$Kg/ms, and graphed the previous equation. In doing that, Rosensweig considered fixed values of $(\frac{m_x}{M_0})$ and calculated the roots for the Equation (4.12). The graph obtained presents a very steep initial rate of change, that Rosensweig expalins is due to the reorientation of the particles in the shear field. The author defined Θ as:

$$\Theta = \arctan\left[\frac{m_x}{M_0}\Big/\left(1 + \frac{m_y}{M_0}\right)\right] \tag{4.13}$$

and with values of $\frac{m_x}{M_0}$ and $\frac{m_y}{M_0}$, calculates values of Θ which are also graphed against $\Omega\tau$. In the graph it can be seen that the value of Θ goes from 0 and approaching to an asymptote value of $\frac{\pi}{2}$. Therefore, in this article the equations of balance angular momentum and magnetization equation were coupled and the shear coefficient of spin viscosity η' was taken in consideration. The conclusion is that, the internal angular momentum and the magnetic field are not aligned due to the shear of viscosity.

In the two articles described above the authors studied a ferrofluid confined between two parallel plates while applying an external magnetic field. We observed that, although using different ways of coupling the balance of angular and linear momentum equations, the results indicate that ferrofluids change the direction of rotation at some point in the channel. In both studies the previous situation was reached when the angular momentum and the linear momentum equations were coupled, considering the anti-symmetric stress boundary condition $2\boldsymbol{\omega} = \nabla \times \mathbf{u}$, that is, angular velocity equals half of the vorticity. This conclusion was the motivation for our own study, which is presented in the next chapter.

Chapter 5

Numerical Results: Non-zero spin viscosity effects

In this chapter we described our study, by introducing the assumptions, non dimensionalize the coupled system of equitations, and derive the coupled system of ordinary differential equations for the translational and spin velocities. We also introduce the appropriate boundary conditions, and present our numerical reusults using MATLAB®.

5.1 Methods

We studied two cases and for both of them, we considered a ferrofluid confined in the gap of a plane Couette cylinder. We analyzed the change in ferrofluid's rotational direction by changing the intensity of the applied magnetic field **H** while maintaining the magnitude of the rotation of the inner cylinder Ω constant. We also did a similar study changing the value of Ω while considering a fixed value of the magnetic field **H**. The balance of linear and angular momentum, and magnetization equations are

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{u}\right)=\nabla\cdot\mathbb{T}+\mathbf{f}, \tag{5.1}$$

$$\rho I\left(\frac{\partial \boldsymbol{\omega}}{\partial t}+\mathbf{u}\cdot\nabla\boldsymbol{\omega}\right)=\nabla\cdot\mathbb{C}+\mathbf{T}+\mathbf{I}, \tag{5.2}$$

and

$$\frac{\partial \mathbf{M}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{M}=\boldsymbol{\omega}\times\mathbf{M}-\frac{1}{\tau}(\mathbf{M}-\mathbf{M}_0). \tag{5.3}$$

The bold quantities represent vectors, and $\mathbb{T}$ and $\mathbb{C}$ tensors; **u** is the translational velocity of the fluid, $\boldsymbol{\omega}$ the spin velocity of the suspension, $\mathbf{M}_0$ the equilibrium of magnetization.

Ferrofluids have particles in suspension, and when we apply an external magnetic field the alignment of the vector magnetization of the the particles with the vector external field will be done through different processes, depending on the size of the particles. These processes take different amount of time. Since we consider ferrofluids as a continuous fluids, we took an average of these different times, calling it *the effective magnetization relaxation time* τ.

The body force density $\mathbf{f} = \mu_0 M \cdot \nabla \mathbf{H}$, the body couple density $\mathbf{L} = \mu_0 M \times H$, $\mathbb{T}$ is the stress tensor, $\mathbb{C}$ the couple stress tensor, and μ_0 the permeability of free space. The expressions of $\mathbb{C}$ and$|tensorT$ are:

$$\mathbb{T} = -p\mathbb{I} + \eta(\nabla \mathbf{u} + \nabla \mathbf{u}^T) + \zeta \mathbb{E} \cdot (\nabla \times \mathbf{u} - 2\boldsymbol{\omega}), \tag{5.4}$$

and

$$\mathbb{C} = \eta'(\nabla \boldsymbol{\omega} + \nabla \boldsymbol{\omega}^T). \tag{5.5}$$

We considered a cylindrical coordinate system (r, Θ, z) and we took all the field quantities as a function of r, the radius of the gap in the plane Couette cylinder. Therefore, we defined the velocity $\mathbf{u} = (0, u(r), 0)$, the spin velocity $\boldsymbol{\omega} = (0, 0, w(r))$, the magnetic field $\mathbf{H} = (h(r), 0, 0)$, and the vector magnetization $\mathbf{M} = (M_1(r), M_2(r), 0)$. The applied magnetic field $\mathbf{H}$ was considered as inversely proportional to the radial distance between cylinders $h(r) = H/r$, where H represents the strength of the magnetic field. We also imposed a vanishing anti-symmetric stress condition $2\boldsymbol{\omega} = \nabla \times \mathbf{u}$, which can be rewritten as:

$$2\boldsymbol{\omega}(r) = \frac{\partial u}{\partial r}(r) + \frac{u(r)}{r + R_1/R} \tag{5.6}$$

and when applied at the boundaries will yield to the boundary conditions for spin.

The components of magnetization can be computed as

$$M_1(r) = \frac{\chi_0 h(r)}{1 + \tau^2 \left(\frac{u(r)}{r} - w(r)\right)^2} \quad \text{and} M_2(r) = \frac{-\chi_0 h(r) \left(\frac{u(r)}{r} - w(r)\right)}{1 + \tau^2 \left(\frac{u(r)}{r} - w(r)\right)^2}. \tag{5.7}$$

For convenience, the parameters were expressed in dimensionless form, which we made by normalizing the space between cylinders as $r = \tilde{r}R + R_1$ with $0 \leq \tilde{r} \leq 1$. The magnetic field normalized as $\tilde{H} = \mathbf{H}/H, \tilde{\mathbf{u}} = \mathbf{u}/U$, the spin velocity normalized as $\tilde{\omega} = \boldsymbol{\omega}/\Omega$. With these parametrizations and using Eq.(5.7), the Eq.(5.2) and Eq.(5.3) can be reduced to:

$$0=(1+\alpha)\left\{u^{''}(r)+\frac{R}{\tilde{r}}u^{'}(r)-\frac{R^2}{\tilde{r}^2}u(r)\right\}-2\alpha w^{'}(r)-\beta h^2\left\{\frac{u(r)R^2-w(r)\tilde{r}R}{\tilde{r}^2+[u(r)R-w(r)\tilde{r}]^2}\right\} \tag{5.8}$$

$$0=\epsilon\left\{w^{''}(r)+\frac{R}{\tilde{r}w^{'}(r)}\right\}-2w(r)+u^{'}(r)+\frac{R}{\tilde{r}}u(r)+\frac{\beta}{\alpha}\tilde{r}h^2(r)\left\{\frac{u(r)R-w(r)\tilde{r}}{\tilde{r}^2+[u(r)R-w(r)\tilde{r}]^2}\right\} \tag{5.9}$$

Where $\epsilon=\frac{\eta'}{R^2\zeta}$ is proportionally related with the shear coefficient of spin viscosity η'. The parameters $\alpha=\frac{\zeta}{\eta}=3\frac{\Phi}{2}$, and $\beta=\frac{\mu_0\chi_0\mathbf{H}^2}{\zeta}$ are dimensionless, hence their quotient is as well.

By solving the Equations (5.8) and (5.9) we were simulating the ferrofluids behavior in a plane Couette Cylinder with a rotating inner cylinder and a fixed outer cylinder while an external magnetic field **H** was applied. The no-slip conditions are related with this assumption $u(0)=\Omega R_1$ and $u(1)=0$. The assumption of non-slip boundary condition, means that at the boundary we consider the fluid to have the same translational velocity as the boundary. Between successive fluid layers exist a deformation proportional to the shear stress. The viscosity of the fluid is the constant of proportionality.

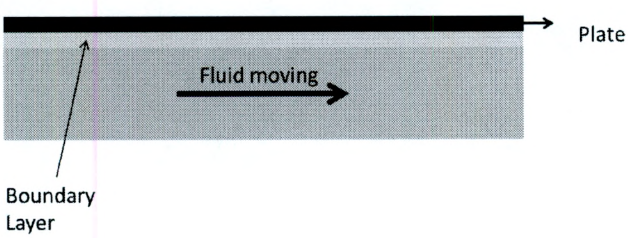

Figure 5.1: *Sketch of a fluid confined between two parallel plates. The upper plate is moving and no-slip boundary conditions are imposed there*

The viscosity is a property that changes directly related with the change in the intensity of the applied magnetic field. Therefore, the more viscous the fluid is, the stronger the shear would be. This will produce a misalignment between the direction of the magnetic field applied and the magnetization vector of the fluid.

We analyzed two cases. First we maintained the value for the external magnetic field **H** constant while applying increasing values to the spin Ω of the inner cylinder. In a second case, we maintained a value of rotation for the inner cylinder and changed the magnitude of the magnetic field. We solved the non-dimensional Equations (5.8) and (5.9) assuming a non-vanishing value of ϵ. In the next section we present the results and the graphs we obtained using MATLAB®.

5.2 Results

The system of coupled nonlinear differential Equations (5.8) and (5.9) helped us to model the flow of a typical ferrofluid between two concentric cylinders under the effect of an external radial magnetic field $\mathbf{H}$. In this section we solved the system, by using the bvpc routine of MATLAB®.

In particular, we assumed that the outer cylinder was fixed and the flow was driven by the combined effects of the rotating inner cylinder and the radial magnetic field applied to the system. For all our studies we considered a small but non-vanishing value of $\epsilon = \frac{\eta'}{R^2\zeta} = \frac{3}{2}\phi$, the ratio of the shear coefficient of spin viscosity (η'), and the product of the square of the radius of the gap of the plane Couette cylinder and the coefficient of the vortex viscosity.

5.2.1 Justification for graphing only for the smallest value of ϵ

We did 2 studies and in both cases we run a MATLAB® program for successively smaller values of ϵ. We observed that, the smaller 2 consecutive values of ϵ were, the more similar the profiles of the spin viscosity $\boldsymbol{\omega} = \boldsymbol{\omega}(r)$ and the translational velocity $\mathbf{u} = \mathbf{u}(r)$ generated for those ϵ were. In the next figures we show this observation.

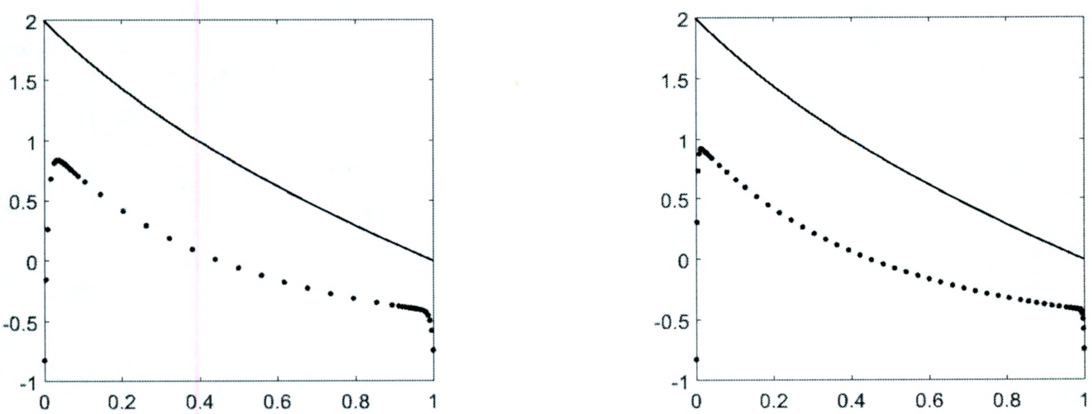

Figure 5.2: *Graphs of profiles of the translational velocity* $\mathbf{u}(r)$ *in bold line, and the angular velocity* $\boldsymbol{\omega}(r)$*in dotted line. The velocity profiles for* $\epsilon = 0.0001$ *are on the left, while those for* $\epsilon = 0.00001$ *are in the right*

Since the profiles observed are very similar, we think there is no need to display all the graphs obtained for all the values of ϵ, then in the next section we present our studies showing profiles obtained only for the smallest values of ϵ.

5.2.2 Constant applied field H and changing rotation rate,Ω, of inner cylinder

When equations were derived, we introduced the parametrization $\mathbf{H} = (h(r), 0, 0)$ with $h(r) = H/r$. There fore the applied magnetic field $\mathbf{H}$ is inversely proportional to the radius of the gap. For the first study-case, we maintained the magnitude of H constantly at 2, while changing the value of the rate of rotation of the inner cylinder Ω. The values we took for this variable were: 0.3, 0.9, 1.2, and 2. Using Eqs. (5.8) and (5.9) and the MATLAB ® program, we generated the graphs shown in Figure (5.2.2). The profiles of the translational velocity $u(r)$ and the angular velocity $\omega(r)$ along the channel are displayed.

The 4 graphs in Figure (5.2.2) illustrate the change of the translational velocity $u(r)$ and the angular velocity $\omega(r)$ when H was constant, the value of the shear coefficient of spin velocity η' was close to zero and 4 different values of the spin of the ferrofluid Ω were assigned.

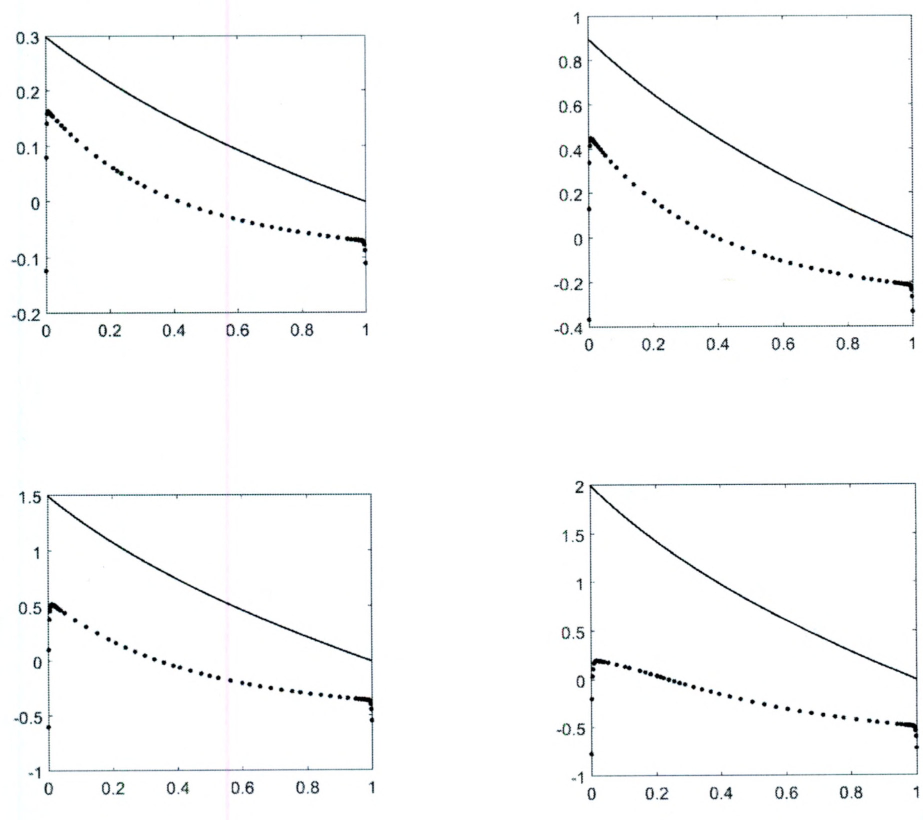

Figure 5.3: *Graphs of the translational velocity* $\mathbf{u}(r)$ *in bold line, and the spin* $\boldsymbol{\omega} = \boldsymbol{\omega}(r)$, *in dotted line. Graphs generated for H=2,* $\Omega = 0.3, \Omega = 0.9, \Omega = 1.5, \Omega = 2,$ *and* $\epsilon = 0.00001$

In all the graphs displayed, we observe that the profile of ω has a change in its sign. That is, a ferrofluid confined in between two parallel plates and under the effect of an imposed radial magnetic field **H**, changes the direction of internal rotation at a certain point of the channel, when the values of the spin change of magnitude. In the next section we present the results of our second study where the value of the rate of rotation of the inner cylinder Ω was maintained constant while the values of the magnetic field changed.

5.2.3 Varying tha applied magnetic field H while the rotation rate Ω is constant

In this section we present the second study-case about the direction of rotation of the ferrofluid. We used the same ferrofluid in between the parallel plates of the plane Couette cylinder, and we applied a radial magnetic field **H**. This time, we gave the values 1, 2, 2.5, and 3 for the magnitude of magnetic field, and we left the rotation rate of the inner cylinder Ω unchanged at 2. Graphs in Figure (5.4) illustrates the change in the profiles of the translational velocity $u(r)$ and in the angular velocity $\omega(r)$ when the spin of the ferrofluid is constant ($\Omega = 2$), the value of the magnetic field H changed and η' was also maintained near zero.

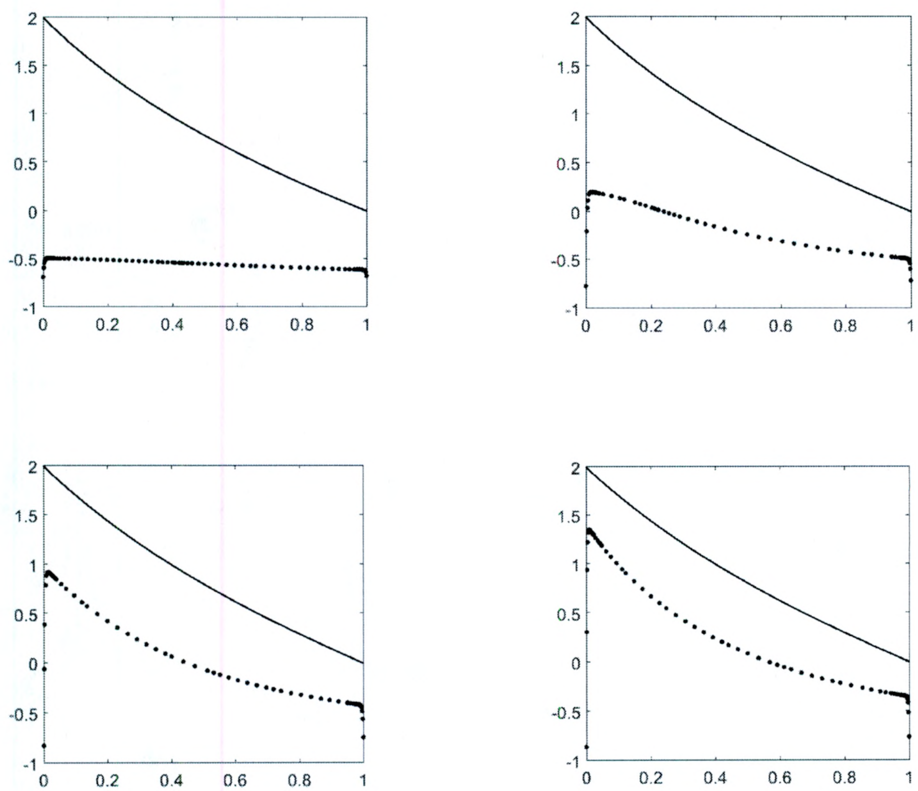

Figure 5.4: *Profile of* $\mathbf{u}(r)$, *in bold line, and* $\boldsymbol{\omega}(r)$, *in dotted line. Graphs generated for* $H = 1, H = 2, H = 2.5, H = 3, \Omega = 2$, *and* $\epsilon = 0.00001$

In all the cases we observe that the direction of rotation of a ferrofluid with a constant value of rotation rate of inner cylinder Ω and changing values of the applied radial magnetic field $\mathbf{H}$, changes at some point of the gap in the plane Couette cylinder.

In the next section we are going to study where the ferrofluid changes its direction of rotation.

5.2.4 Location in the channel where the spin velocity $\omega(r)$ reverses direction

In the previous sections, we presented our 2 study cases. A ferrofluid in a plane Couette cylinder to which is applied a radial magnetic field $\mathbf{H}$. In the first case we maintained the magnitude of the magnetic field unaltered while changing the values of the rate of rotation of the inner cylinder Ω. For the second case we changed the values of the magnetic field $\mathbf{H}$ applied while maintaining fixed the value of therate of the rotation of the inner cylinder Ω. In both circumstances, we observed that the direction of the internal rotation of the ferrofluid changes at some point in the channel. We defined the variable r_c as the distance between the inner cylinder, and the point of the channel at where the change in the direction of internal rotation takes place. Our computed values for r_c are presented Tables (5.1) and (5.2).

Table 5.1: *Computed values of r_c, the location of spin reversal in the cylinder for constant field magnetic field H and $\Omega = 0.3, \Omega = 0.9, \Omega = 1.5,$ and $\Omega = 0.3$*

H	Ω	r_c
2	0.3	0.4205
2	0.9	0.4004
2	1.5	0.3479
2	2	0.2417

From Table(5.1), we observe that exists an inverse relationship between the magnitude of the angular velocity $\Omega = 0.9$ and the place r_c of the channel where the internal rotation reverses rotation.

From Table (5.2) we observe a direct relationship between the magnitude of the applied magnetic field $\mathbf{H}$ and the location of the channel at where the fluid changes direction of interanl rotation. We applied a radial magnetic field to a ferrofluid in a plane Couette cylinder. We studied two cases: first we maintained the magnitude of the magnetic field H constant and changed the value of the velocity Ω of the

Table 5.2: *Computed values of r_c, the location of the spin reversal in the cylinder for constant value of $\Omega = 2$ while the applied magnetic field changes H=1, H=2, H=2.5, and H=3*

H	Ω	r_c
1	2	No changes
2	2	0.2417
2.5	2	0.4509
3	2	0.565

inner cylinder. For the second case we changed the value of the magnetic field and left constant the value rotation rate of the inner cylinder. The graphs and tables presented in Sections 5.2.2, 5.2.3 and 5.2.4, show that in both cases the direction of the internal rotation of the ferrofluid changes at some point in the channel. The analysis described, was done only for four values of magnetic field and spin. Future studies may be done considering the asymptotic limit for this variables.

Chapter 6

Discussion and Conclusions

We studied the case of a ferrofluid in a plane Couette cylinder under the effect of a radial magnetic field under 2 different conditions. For the first case, we left constant the value of the magnetic field $\mathbf{H}$, while changing the rate of rotation of the inner cylinder Ω. In the second study, we varied the value of the magnetic field H and left unchanged the magnitude of the rate of rotation of the inner cylinder Ω. The equations for the balance of linear and angular momentum for ferrofluids were supplemented with the non-slip boundary conditions $u(0) = \Omega R_1$ and $u(1) = 0$ for the translational velocity, and the vanishing antisymetric stress condition $2\boldsymbol{\omega} = \nabla \times \mathbf{u}$ for the spin velocity. Our numerical studies for this model predict that the internal rotation of the ferrofluid changed the direction at a certain point in the channel.

This conclusion concurs with the findings of Rosensweig [14], and Jothimani and Devi [5]. Although the assumptions and the way of manipulating the equations in those articles were different than in ours, still a change in the direction of the rotational spin was observed. The most relevant similarity between the studies, is considering a realistic non-zero value for the shear coefficient of spin viscosity. Our findings agree with the studies of Jothimani and Devi [5] who used different kind of magnetic fields. In their study, the authors made certain assumptions and solved a model which is essentially similar top ours. They did the same study for an axial magnetic field, a transverse magnetic field, and a rotating magnetic field, and solved for different cases. In one of them, the authors considered the spin-viscosity (η') as nonzero with zero magnetic torque density [14]. As a result, once the equations are coupled and the spin viscosity is assumed as non-zero, a change in the direction of the angular velocity of the fluid is observed at some point in the channel. Their findings, along with ours, suggest that the observed change in direction of the spin velocity of the fluid is due viscous property of the fluid.

A similar conclusion is presented by Rosensweig, where the term $\nabla \mathbf{C}$ is neglected (see discussion in [11]). This term, which accounts for the influence of η', represents transport of angular momentum by diffusion of spinning magnetic particles down a gradient of spin [11]. In his article, Rosensweig simulates the behavior of a ferrofluid under the influence of oscillatory and rotating magnetic fields. The author shows that, the angle between the externally magnetic field applied and the internal angular momentum increases along the width of the channel.

The findings of the present study contribute further evidence to the change in rotational spin of ferrofluid particles. Future research should focus in the influence of different variables that contribute to this phenomenon and its implications to multi-disciplinary applications.

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